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Cyclic homology of affine hypersurfaces with isolated singularities

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Abstract

We consider reduced, affine hypersurfaces with only isolated singularities. We give an explicit computation of the Hodge-components of their cyclic homology in terms of de Rham cohomology and torsion modules of differentials for large n. It turns out that the vector spaces $\mathrm{HC}_n(A)$ are finite dimensional for $n \geq N-1$. © 1997 Elsevier Science B.V.

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1. Introduction

Let $R = K[X_1, X_2, \ldots, X_N]$ with K being an algebraically closed field of characteristic zero. Throughout this paper A denotes a reduced hypersurface with only isolated singularities given by $A = R/(F)$, whith $F \in K[X_1, X_2, \ldots, X_N]$. For a definition of the module of Kähler differentials $\Omega^1_{A/K}$ see for example [22, 8.8.1., p. 294]. The cohomology of the complex

$$0 \to A \xrightarrow{d} \Omega^1_{A/K} \xrightarrow{d} \Omega^2_{A/K} \xrightarrow{d} \cdots \xrightarrow{d} \Omega^N_{A/K} \to 0 \to \cdots \to 0,$$

where d denotes the exterior differential, is called the de Rham cohomology of A and denoted by $H^i_{\mathrm{dR}}(A)$. In this paper we will compute the Hodge components $\mathrm{HC}^{(i)}_n(A)$ of cyclic homology of a hypersurface with isolated singularities. We will use the identification in [20] of the nth Hochschild homology groups with torsion submodules of differentials. Throughout this paper the torsion submodule of the $(N-1)$st exterior power of the Kähler differentials will be denoted by $T(\Omega^{N-1}_{A/K})$.

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Theorem 1. *Let A be a reduced, affine hypersurface over K with only isolated singularities and let $n > N$. Then the Hodge-components of cyclic homology are given by:*

$$\mathrm{HC}_n^{(i)}(A) \simeq \begin{cases} T(\Omega^{N-1}_{A/K}) \oplus \boldsymbol{H}^{N-1}_{\mathrm{dR}}(A) & \text{if } 2i-n = N-1, \\ \boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A) & \text{otherwise.} \end{cases}$$

Summing up the Hodge-components, we get

$$\mathrm{HC}_n(A) \simeq \boldsymbol{H}^{N}_{\mathrm{dR}}(A) \oplus \boldsymbol{H}^{N-2}_{\mathrm{dR}}(A) \oplus \cdots \quad \text{if } n \equiv N \bmod 2,$$

$$\mathrm{HC}_n(A) \simeq T(\Omega^{N-1}_{A/K}) \oplus \boldsymbol{H}^{N-1}_{\mathrm{dR}}(A) \oplus \boldsymbol{H}^{N-3}_{\mathrm{dR}}(A) \oplus \cdots \quad \text{if } n \equiv N-1 \bmod 2.$$

Hence we obtain a similar formula as obtained in the smooth case by Loday and Quillen in [15] with an extra nonzero term $T(\Omega^{N-1}_{A/K})$ appearing for $n \geq N$. The shift operator S, see [14] for a definition,

$$S : \mathrm{HC}^{(N+l)}_{N+2l+1}(A) \mapsto \mathrm{HC}^{(N+l-1)}_{N+2l-1}(A)$$

is no longer an isomorphism, see Corollary 1. For quasi-homogeneous hypersurfaces with isolated singularities Theorem 1 and Proposition 1 were already proved in [18]. In the final section we demonstrate the algorithmic nature of our results by computing the cyclic homology and the de Rham cohomology of the nodal cubic.

Remark. It is an immediate consequence of Theorem 1 and Lemma 1 below that for $n \geq N-1$ all the cyclic homology groups $\mathrm{HC}_n(A)$ are finite dimensional K vector spaces.

For $n \leq N$ we have:

Proposition 1 (cf. Michler [18, Theorem 2]). *For $n \leq N$ we compute $\mathrm{HC}_n(A)$ for reduced hypersurfaces A with only isolated singularities*:

$$\mathrm{HC}_n^{(i)}(A) \simeq \begin{cases} \dfrac{\Omega^n_{A/K}}{\mathrm{d}\Omega^{n-1}_{A/K}} & \text{if } i = n, \\ \boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A) & \text{for } n/2 \leq i < n, \\ 0 & \text{otherwise.} \end{cases}$$

2. Hodge-components of Hochschild and cyclic homology

In [8] Gerstenhaber and Schack obtained a Hodge-decomposition

$$\mathrm{HH}_n(A) = \mathrm{HH}_n^{(1)}(A) \oplus \cdots \oplus \mathrm{HH}_n^{(n)}(A)$$

of the Hochschild homology $\mathrm{HH}_n(A)$ of a commutative K-algebra A, where K is a field of characteristic zero. From [18, Theorem 1 and Lemma 1] we know:

Proposition 2. *Let A be the coordinate ring of a reduced hypersurface of dimension $N-1$ with only isolated singularities, then for $n \geq N$ the Hodge-components of Hochschild homology are given by*

$$\mathrm{HH}_n^{(i)}(A) \simeq \begin{cases} T(\Omega_{A/K}^{N-1}) & \text{if } 2i-n=N-1, \\ \Omega_{A/K}^{N} & \text{if } 2i-n=N, \\ 0 & \text{otherwise.} \end{cases}$$

For $n < N$ we have

$$\mathrm{HH}_n^{(i)}(A) \simeq \begin{cases} \Omega_{A/K}^{n} & \text{if } i=n, \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3 (Michler [20]). *Let $F \in K[X_1,\dots,X_N]$ be a polynomial defining a reduced hypersurface with only isolated singularities in A_K^N, where K is an algebraically closed field of characteristic zero. Then we have*

$$\dim_K T(\Omega_{A/K}^{N-1}) = \dim_K \Omega_{A/K}^{N},$$

where $T(\Omega_{A/K}^{N-1})$ is the torsion submodule of $\Omega_{A/K}^{N-1}$, the $(N-1)$st exterior power of the module of Kähler differentials. In particular we have, for $l > 0$,

$$\dim_K \mathrm{HH}_{N+2l}^{(N+l)} = \dim_K \mathrm{HH}_{N+2l-1}^{(N+l-1)} = \dim_K T(\Omega_{A/K}^{N-1}) = \dim_K \Omega_{A/K}^{N} < \infty,$$

and all other Hodge-components of Hochschild homology are zero.

We use the Hodge-decomposition of Hochschild homology to determine the Hodge-components of cyclic homology (cf. [14]):

$$\mathrm{HC}_n(A) = \mathrm{HC}_n^{(1)}(A) \oplus \mathrm{HC}_n^{(2)}(A) \oplus \ldots \oplus \mathrm{HC}_n^{(n)}(A).$$

There is also a long exact $S-B-I$ sequence [13],

$$\cdots \to \mathrm{HH}_n^{(i)}(A) \xrightarrow{I} \mathrm{HC}_n^{(i)}(A) \xrightarrow{S} \mathrm{HC}_{n-2}^{(i-1)}(A) \xrightarrow{B} \mathrm{HH}_{n-1}^{(i)}(A) \xrightarrow{I} \cdots.$$

We will also need the following result (cf. [3, Theorem 3.17]):

Lemma 1. *Let A be the coordinate ring of a reduced affine hypersurface with only isolated singularities defined over an algebraically closed field K of characteristic zero, then the de Rham cohomology groups $\boldsymbol{H}_{\mathrm{dR}}^i(A)$ are finite dimensional K-vector spaces.*

Proof. Let $X = \mathrm{Spec}(A)$ be the hypersurface in question. Replacing complex analytic space by algebraic variety over K and the reference to Grauert in the proof by [9, 3.2.1] in [3, Theorem 3.17], we get: Let X be an algebraic variety over K and x an isolated singular point. Then the cohomology groups of the complex

$$0 \to K \to \Omega_{X,x}^{*}$$

are finite dimensional K-vector spaces. Next we note that $\boldsymbol{H}^i_{\mathrm{dR}}(A)$ is the global sections of the (quasi-coherent) sheaf $\mathscr{H}^i_{\mathrm{dR}}$ on $\mathrm{Spec}(K)$. If we denote the singular locus by Z and $X - Z$ by U, then for all $i \geq 0$ we get an exact sequence of sheaves:

$$0 \to \mathscr{H}^0_Z(\mathscr{H}^i_{\mathrm{dR}}) \to \mathscr{H}^i_{\mathrm{dR}} \to \mathscr{H}^i_{\mathrm{dR}}|_U \to 0.$$

The result now follows on taking global sections, since by [10] the $H^0(\mathscr{H}^i_{\mathrm{dR}}|_U)$ are finite dimensional vector spaces and by [3] the $H^0_Z(\mathscr{H}^i_{\mathrm{dR}})$ are also finite dimensional.

3. Proof of the main theorem

In this section we let A, K be as in the introduction. The following Lemmas 2, 4, 5 and Corollary 1 will establish Theorem 1:

Lemma 2. *If* $0 \leq 2i - n \leq N - 3$ *or* $2i - n \geq N + 1$ *and* $n > N$, *then*

$$\mathrm{HC}^{(i)}_n(A) \simeq \boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A).$$

Moreover, for $2i - n \geq N + 1$ *we have* $\mathrm{HC}^{(i)}_n(A) = 0$.

Proof. From the $S - B - I$ sequence and our computation of the Hodge-components of Hochschild homology we see

$$\mathrm{HC}^{(i)}_n(A) \simeq \mathrm{HC}^{(i-1)}_n(A) \quad \text{for } 2i - n \leq N - 3 \text{ or } 2i - n \geq N + 1.$$

Write $n = N + 2l$, then we have

$$\mathrm{HC}^{(i)}_n(A) \simeq \mathrm{HC}^{(i-l)}_N(A) \simeq \boldsymbol{H}^{2i-2l-N}_{\mathrm{dR}}(A) = \boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A).$$

If $n = N + 2l - 1$ then

$$\mathrm{HC}^{(i)}_n(A) \simeq \mathrm{HC}^{(i-l)}_{N-1}(A) \simeq \boldsymbol{H}^{2i-2l-N+1}_{\mathrm{dR}}(A) = \boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A).$$

For $2i - n \geq N + 1$ we have that $\boldsymbol{H}^{2i-n}_{\mathrm{dR}}(A) = 0$, since A is affine. □

The key ingredient in the proof is the following lemma:

Lemma 3. *Let* A *be as before and assume* $2i - n = N$ *with* $n \geq N$, *then the map* $B : \mathrm{HC}^{(i-1)}_{n-1}(A) \to \mathrm{HH}^{(i)}_n(A)$ *is surjective.*

Proof. If $i = n = N$, then we know that $\mathrm{HC}^{(N-1)}_{N-1}(A) \simeq \Omega^{N-1}_{A/K}/\mathrm{d}\Omega^{N-2}_{A/K}$ and since $d : \Omega^{N-1}_{A/K} \to \Omega^N_{A/K}$ is surjective, the result follows by [15, Proposition 2.2]. By [12] we have, for $i > N$,

$$\mathrm{HH}^{(i)}_n(A) = H^{2i-n}(\mathrm{Kos}^*(F/F^2 \to A \otimes \Omega^1_{R/K})_i),$$

where the ith homogeneous part of the Koszul-complex is given by

$$\mathrm{Kos}^*(F/F^2 \to A \otimes \Omega^1_{R/K})_i:$$

$$0 \leftarrow \frac{F^{i-N}}{F^{i-N+1}} \otimes \Omega^N_{R/K} \leftarrow \left(\frac{F^{i-N+1}}{F^{i-N+2}}\right) \otimes \Omega^{N-1}_{R/K} \leftarrow \cdots \leftarrow \left(\frac{F^i}{F^{i+1}}\right) \otimes R \leftarrow 0.$$

Moreover, Feigin and Tsygan [6] show that

$$0 \to \mathrm{Kos}^* \to D^* \to D^{*-1} \to 0,$$

where the complex $(D^*)_i$ is given by

$$\cdots \leftarrow 0 \leftarrow \Omega^N_{R/K}/F^{i-N+1}\Omega^N_{R/K} \leftarrow \Omega^{N-1}_{R/K}/F^{i-N+2}\Omega^{N-1}_{R/K} \leftarrow \cdots \leftarrow R/F^{i+1}R \leftarrow 0.$$

The cohomology of this complex computes the ith level crystalline cohomology (cf. [6].) We then get the following long exact sequence on homology, see [16]:

$$\to H^{N-1}(\mathrm{Kos}^*_{N+l-1}) \to H^{N-1}(D^*_{N+l}) \to H^{N-1}(D^*_{N+l-1}) \to H^N(\mathrm{Kos}^*_{N+l}) \to$$

$$H^{N-1}(D^*_{N+l-1}) = \frac{\mathrm{Ker}(\Omega^{N-1}_{R/K}/F^{l+1}\Omega^{N-1}_{R/K} \to \Omega^N_{R/K}/F^l\Omega^N_{R/K})}{\mathrm{Im}(\Omega^{N-2}_{R/K}/F^{l+2}\Omega^{N-2}_{R/K} \to \Omega^{N-1}_{R/K}/F^{l+1}\Omega^{N-1}_{R/K})}.$$

In [6] Feigin and Tsygan define K-vector space isomorphisms ϕ, ψ for all indices n. From their Theorem 5 [6, p.130], we get that for any positive integer l the following diagram commutes:

$$\begin{array}{ccccccc}
\mathrm{HH}^{(N+l)}_{N+2l+1}(A) & \xrightarrow{I} & \mathrm{HC}^{(N+l)}_{N+2l+1}(A) & \xrightarrow{S} & \mathrm{HC}^{(N+l-1)}_{N+2l-1}(A) & \xrightarrow{B} & \mathrm{HH}^{(N+l)}_{N+2l}(A) \\
\simeq \downarrow \psi & & \simeq \downarrow \phi & & \simeq \downarrow \phi & & \simeq \downarrow \psi \\
H^{N-1}(\mathrm{Kos}^*_{N+l-1}) & \longrightarrow & H^{N-1}(D^*_{N+l}) & \longrightarrow & H^{N-1}(D^*_{N+l-1}) & \xrightarrow{\varepsilon} & H^N(\mathrm{Kos}^*_{N+l})
\end{array}$$

In the above long exact sequence $\varepsilon : H^{N-1}(D^*_{i-1}) \to H^N(\mathrm{Kos}^*_i)$ is the connecting homomorphism. Since the Kähler map $d : \Omega^{N-1}_{R/K} \to \Omega^N_{R/K}$ is surjective, the induced map

$$\tilde{d} : \Omega^{N-1}_{R/K}/F^{k+1}\Omega^{N-1}_{R/K} \to \Omega^N_{R/K}/F^k\Omega^N_{R/K} \quad \textit{for all integers } k > 0,$$

is surjective as well. Hence $H^N(D^*_{N+l})$ is zero and the result follows by [6, Theorem 5]. □

Let $2i - n = N$ with $n \geq N$ and consider the following sequence, which is exact by Proposition 2:

$$(*) \qquad 0 \to \mathrm{HC}^{(i)}_{n+2}(A) \xrightarrow{S} \mathrm{HC}^{(i-1)}_n(A) \xrightarrow{B} \mathrm{HH}^{(i)}_{n+1}(A) \xrightarrow{I} \mathrm{HC}^{(i)}_{n+1}(A) \xrightarrow{S}$$
$$\xrightarrow{S} \mathrm{HC}^{(i-1)}_{n-1}(A) \xrightarrow{B} \mathrm{HH}^{(i)}_n(A) \xrightarrow{I} \mathrm{HC}^{(i)}_n(A) \xrightarrow{S} \mathrm{HC}^{(i-1)}_{n-2}(A) \to 0.$$

Corollary 1. *Let* $2i - n = N$ *with* $n \geq N$ *as before, then*:
(a) $\mathrm{HC}_n^{(i)}(A) = 0$.
(b) $I : \mathrm{HH}_n^{(i)}(A) \to \mathrm{HC}_n^{(i)}(A)$ *is the zero map.*
(c) *if* $n > N$ *then the map* $S : \mathrm{HC}_{n+1}^{(i)}(A) \to \mathrm{HC}_{n-1}^{(i-1)}(A)$ *is not surjective.*

Proof. Note that we have $\mathrm{HC}_N^{(N)}(A) = \boldsymbol{H}_{\mathrm{dR}}^N(A) = 0$. The results then follow from the exactness of the part $(*)$ of the $S{-}B{-}I$ sequence, Lemma 3 and induction. In particular we have that: $\mathrm{coker}(S : \mathrm{HC}_{n+1}^{(i)}(A) \to \mathrm{HC}_{n-1}^{(i-1)}(A)) \simeq \Omega_{A/K}^N \neq 0$.

Lemma 4. *If* $2i - n = N$ *with* $n \geq N$ *then,* $B : \mathrm{HC}_n^{(i-1)}(A) \to \mathrm{HH}_{n+1}^{(i)}(A)$ *is the zero map and we have*:

$$\mathrm{HC}_{n+2}^{(i)}(A) \simeq^S \mathrm{HC}_n^{(i-1)}(A) \simeq \boldsymbol{H}_{\mathrm{dR}}^{N-2}(A).$$

In particular $\mathrm{HC}_n^{(i)}(A)$ *is finite dimensional.*

Proof. From Proposition 1 we know that $\mathrm{HC}_N^{(N-1)}(A) = \boldsymbol{H}_{\mathrm{dR}}^{N-2}(A)$. By [6] we have that

$$\mathrm{HC}_{N-2+2l}^{(N-2+l)}(A) \simeq H^{N-2}(D_{N-2+l}^*)$$

for all $l \geq 1$. Moreover, from $(*)$ and induction on i we know that the S maps $S : \mathrm{HC}_{n+2}^{(i)}(A) \mapsto \mathrm{HC}_n^{(i-1)}(A)$ are injections. Hence we get injections from $\mathrm{HC}_{n+2}^{(i)}(A)$ into $\boldsymbol{H}_{\mathrm{dR}}^{N-2}(A)$.

On the other hand, we know by [2, Theorem 5(b)] that $\dim_K H^{N-2}(D_{N-2+l}^*) \geq \dim_K H_{\mathrm{dR}}^{N-2}(R/F^l R)$. Since F is reduced we have by [22, Exercise 9.9.5 p. 359] that $H_{\mathrm{dR}}^{N-2}(R/F^l R) = H_{\mathrm{dR}}^{N-2}(A)$. Hence we have for all $l \geq 1$ that $\mathrm{HC}_{N-2+2l}^{(N-2+l)}(A) = \boldsymbol{H}_{\mathrm{dR}}^{N-2}(A)$. □

Corollary 2. *Again let* $2i - n = N$ *with* $n \geq N$, *then*:
(a) *The map* $I : \mathrm{HH}_{n+1}^{(i)}(A) \to \mathrm{HC}_{n+1}^{(i)}(A)$ *is injective.*
(b) $\dim_K \mathrm{HC}_n^{(i-1)}(A) < \infty$.

Lemma 5. *Let* $2i - n = N$ *with* $n \geq N$, *then*:
(a) $\dim_K \mathrm{HC}_{n-1}^{(i-1)}(A) = \dim_K \boldsymbol{H}_{\mathrm{dR}}^{N-1}(A) + \dim_K T(\Omega_{A/K}^{N-1})$.
(b) $\mathrm{Ker}(B : \mathrm{HC}_{n-1}^{(i-1)}(A) \to \mathrm{HH}_n^{(i)}(A)) \simeq \boldsymbol{H}_{\mathrm{dR}}^{N-1}(A)$.

Proof. By Lemmas 3 and 4 we can split up $(*)$ as follows:

$$0 \to \mathrm{HH}_{n+1}^{(i)}(A) \xrightarrow{I} \mathrm{HC}_{n+1}^{(i)}(A) \xrightarrow{S} \mathrm{Im}(S) \to 0,$$

and

$$0 \to \mathrm{Ker}(B) \to \mathrm{HC}_{n-1}^{(i-1)}(A) \xrightarrow{B} \mathrm{HH}_n^{(i)}(A) \to 0,$$

where $\mathrm{Ker}(B) = \mathrm{Im}(S)$. By induction on i we then know that all $\mathrm{HC}_{n+1}^{(i)}(A)$'s have the same dimension. If $i = n = N$ then by [15] we have $\mathrm{Ker}(B : \mathrm{HC}_{N-1}^{(N-1)}(A) \to$

$\mathrm{HH}_N^{(N)}(A)) \simeq \boldsymbol{H}_{\mathrm{dR}}^{N-1}(A)$. (Note that both $T(\Omega_{A/K}^{N-1})$ and $\boldsymbol{H}_{\mathrm{dR}}^{N-1}(A)$ are finite dimensional vector spaces.) Hence we have that

$$\dim_K \mathrm{HC}_{N-1}^{(N-1)}(A) = \dim_K \frac{\Omega_{A/K}^{N-1}}{d\Omega_{A/K}^{N-2}} = \dim_K \boldsymbol{H}_{\mathrm{dR}}^{N-1}(A) + \dim_K \Omega_{A/K}^{N}$$
$$= \dim_K T(\Omega_{A/K}^{N-1}) + \dim_K \boldsymbol{H}_{\mathrm{dR}}^{N-1}(A),$$

where the last equality follows from Proposition 3. □

Corollary 3. *If A is a hypersurface with only isolated singularities over an algebraically closed field K of characteristic zero, then for $n \geq N-1$ we have that the nth pieces of cyclic homology are finite dimensional vector spaces. Moreover, the dimension of* $\mathrm{HC}_n(A)$ *only depends on the parity of n for $n \geq N$.*

4. Cyclic homology of the nodal cubic

This section contains an explicit description of the Hodge-components of the plane nodal cubic defined by $F(x, y) = y^2 - x^2(x+1)$, i.e.,

$$A = K[x, y]/(y^2 - x^3 - x^2).$$

In particular we give bases for its de Rham cohomology modules. This example was also mentioned in [7, cf. A.6], where a different approach to finding $\mathrm{HC}_n(A)$ was suggested. We have:

Proposition 4. *The de Rham cohomology of the nodal cubic is given by*

$$\boldsymbol{H}_{\mathrm{dR}}^0(A) = K, \qquad \boldsymbol{H}_{\mathrm{dR}}^1(A) \simeq K \cdot x^2 \mathrm{d}y + \mathrm{d}A, \qquad \boldsymbol{H}_{\mathrm{dR}}^2(A) = 0.$$

Proof. The coordinate ring of the nodal cubic is a domain. So we know that $\boldsymbol{H}_{\mathrm{dR}}^0(A) = K$. Moreover, $\Omega_{A/K}^2 = K\mathrm{d}x \wedge \mathrm{d}y$, i.e., $\boldsymbol{H}_{\mathrm{dR}}^2(A) = 0$. Using the fact that

$$y\mathrm{d}y = \frac{(3x^2 + 2x)}{2}\mathrm{d}x$$

we have that

$$\frac{\Omega_{A/K}^1}{\mathrm{d}A} = \frac{(K \cdot x\mathrm{d}y + K \cdot x^2\mathrm{d}y + \mathrm{d}A)}{\mathrm{d}A}.$$

Hence

$$\boldsymbol{H}_{\mathrm{dR}}^1(A) = \mathrm{Ker}(\tilde{d} \,:\, \Omega_{A/K}^1/\mathrm{d}A \to \Omega_{A/K}^2) = (K \cdot x^2\mathrm{d}y + \mathrm{d}A)/\mathrm{d}A.$$

A generator for $T(\Omega_{A/K}^1)$ as an A-module is given by

$$w = (\tfrac{3}{2}x + 1)y\,\mathrm{d}x - (x^2 + x)\,\mathrm{d}y.$$

We sum up our findings in:

Theorem 2. *Let A be the coordinate ring of the nodal cubic defined over an algebraically closed field K of characteristic zero by* $F = y^2 - x^2(x+1)$, *then* $\mathrm{HC}_0^{(0)}(A) = A$ *and we have, for* $l > 0$,

$$\mathrm{HC}_{2l}(A) = \boldsymbol{H}^0_{\mathrm{dR}}(A) = K,$$

and

$$\mathrm{HC}_{2l+1}(A) \simeq T(\Omega^1_{A/K}) \oplus \boldsymbol{H}^1_{\mathrm{dR}}(A) \simeq K \oplus K.$$

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